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Thermal Similitude Studies

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One of the initial objectives of work in thermal similitude is to determine modeling laws and techniques which will find application in thermal scaling of spacecraft and in research problems. Modeling laws for space vehicles are derived from the differential equations used in practice for thermal analysis. The necessary and sufficient conditions for complete thermal similarity between model and prototype take the form of 28 ratios that must remain constant. However, all of these ratios are not independent but contain independent sets of six ratios. An example of thermal scaling is given in which one independent set is used. The general results compare favorably with those of some investigators who used other methods. The purposes of full-scale thermal testing are discussed, and the usefulness of thermal scale modeling as a substitute for full-scale testing is questioned. It is suggested that the principles of similarity and scale modeling may find the most fruitful application in special research problems.

Nomenclature

C_j = heat capacity of jth region, kcal/°K
T_j = temperature of jth region, °K
T_k = temperature of kth region, °K
$\dot{T}_j$ = derivative of T_j with respect to time, °K/hr
C_{kj} = Over-all conduction coefficient between regions k and j, kcal/hr-°K
R_{kj} = over-all radiant coefficient for net radiative transfer from region k to j, kcal/hr (°K)
q_j = internal power dissipation, kcal/hr
${}^sA_j$ = projected area of surface j to sun, m²
A_j = total area of surface j radiating to space, m²
ϵ_j = infrared emissivity of surface j, dimensionless
α_j = absorptivity of surface j with respect to solar insulation, dimensionless
S = solar constant, 1200 kcal/hr m²
σ = Stefan-Boltzmann constant, 4.87×10^{-8} kcal/hr m², (°K)⁴ (β_j, γ_j, δ_j, η_j, θ, ζ_j, ι_j, λ_j, ω, μ_{kj}, ρ_{kj} are constants of proportionality, as indicated in text)
b_j = particular values of T_j at zero time
F_{kj} = classical geometry factor for net radiation exchange based on Lambert's cosine law for diffuse radiation
E_{kj} = factor in over-all radiation coefficient that depends only on emissivities of surfaces for bodies k and j
R_{A_k} = area for radiation coupling term, m²
l_2, d_1, d_2, h, a, L_1 = particular linear dimensions, m
V_j = volume of section j, m³
k_j = thermal conductivity of jth region, kcal/hr-m-°K
$(\rho c_p)_j$ = density-specific heat product for jth region (volumetric heat capacity), kcal/m³ °K
$(\rho c_p)_N$ = volumetric heat capacity normalized to 6061 alloy $(\rho c_p)_{Al}/(\rho c_p)_j$
k_N = thermal conductivity normalized KEL-F, $k_j/k_{\text{KEL-F}}$
ω_2 = solid angle subtended by A_2, as viewed from dA_1
ϕ_1 = angle between normal to dA_1 and line of sight to differential area A_2

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Introduction

THE problems of interest in the field of thermal similitude can be separated into two categories: 1) those for which a mathematical model in the form of descriptive differential equations is available, and 2) those for which adequate governing equations are unknown. Most of this paper is devoted to consideration of the former category.

Similarity Based on n Simultaneous First-Order Differential Equations

The following set of n differential equations may be used to describe the temperature behavior of n bodies that are at different but uniform temperatures:

$$C_j\dot{T}_j = \sum_{\substack{k=1\\k\neq j}}^{n} C_{kj}(T_k - T_j) + \sum_{\substack{k=1\\k\neq j}}^{n} R_{kj}(T_k{}^4 - T_j{}^4) + q_j + {}^sA_j\alpha_jS - A_j\epsilon_j\sigma T_j{}^4 \qquad j = 1, \ldots, n \quad (1)$$

Systems of equations of this form are widely used to predict the temperatures of regions of satellites and other space vehicles.[1,2] The method has been used successfully for the past six years in the thermal design of earth satellites.[3,4] It consists in dividing the vehicle into n regions that can be considered isothermal and then writing a thermal energy balance for each region. The terms in the general equation given here, reading from left to right, are heat capacity/time response, two coupling terms (one for conduction and one for radiation), internal source, direct solar insulation, and radiation from the body to space. In the general case for an earth satellite there would occur also terms for earth albedo and infrared radiation. However, these were omitted because it is standard practice in thermal testing to include only solar simulation or some other equally effective external heating and internal equipment dissipation.[5]

There is no fundamental conceptual difficulty in the use of these equations, since any region can be further subdivided until the assumption of isothermal sections is close enough. In practical terms, the finer the subdivision, the greater the number of equations that must be solved on the computer, the more work required to determine load-in data, and the greater computer-storage and computation-time requirements. Another interesting assumption implied by the equations is that the conduction and radiation transfer between regions is linearly independent. This is approximately true if there is not significant radiation to the conducting path.† Situations can occur in which this technique leads to an excessive number of equations if diligently applied. This particular problem is currently under separate study and will not be discussed further here.

These differential equations (1) are taken as the descriptive mathematical model of the thermal prototype. The corresponding set of equations for a thermally similar model would be

$$C_j{}^*\dot{T}_j{}^* = \sum_{\substack{k=1\\k\neq j}}^{n} C_{kj}{}^*(T_k{}^* - T_j{}^*) + \sum_{\substack{k=1\\k\neq j}}^{n} R_{kj}{}^*(T_k{}^{*4} - T_j{}^{*4}) + q_j{}^* + {}^SA_j{}^*\alpha_j{}^*S^* - A_j{}^*\epsilon_j{}^*\sigma T_j{}^{*4} \qquad j = 1, \ldots, n \quad (2)$$

The objective is to determine the necessary and sufficient conditions for the existence of scale factors, or similarity, such that the following relations hold:

$$\begin{array}{llll} T_j = \beta_j T_j{}^* & A_j = \zeta_j A_j{}^* & & \\ q_j = \gamma_j q_j{}^* & \epsilon_j = \iota_j \epsilon_j{}^* & & \\ {}^SA_j = \delta_j {}^SA_j{}^* & C_j = \lambda_j C_j{}^* & & \\ \alpha_j = \eta_j \alpha_j{}^* & t = \omega t^* & & \\ S = \theta S^* & j = 1, \ldots, n & & (3) \end{array}$$

$$\left.\begin{array}{l} C_{kj} = \mu_{kj} C_{kj}{}^* \\ R_{kj} = \rho_{kj} R_{kj}{}^* \end{array}\right\} \begin{array}{l} j = 1, \ldots, n \\ k = 1, \ldots, n;\ k \neq j \end{array}$$

The demand represented by Eq. (3) is that each of the variables and parameters occurring in the differential equations for the prototype correspond with those for the model through simple proportion. Briefly, it does not represent simply a reduction of all linear dimensions by some single constant scale factor but, instead, includes the possibility of circumventing practical difficulties in the detailed design of models—that is, it allows the possibility of distorting the model and still retaining essential similarity. This is the main advantage of the differential method that will be used—a well-known method given in standard references.[6, 7]

† For example, it happens in some structures that two surfaces which exchange energy by radiation are connected by a structural member that provides a conducting path between the two surfaces, while the surface of the member also exchanges radiant energy with the two surfaces.

Substituting the relations of Eq. (3) into the set of Eqs. (2) and comparing the resulting set of differential equations with the set of Eqs. (1), term for term, the following relations among the scale factors are obtained:

$$\begin{gathered} \lambda_j\beta_j/\omega = \delta_j\eta_j\theta = \zeta_j\iota_j\beta_j{}^4 = \gamma_j = \mu_{1j}\beta_1 = \mu_{2j}\beta_2 = \\ \ldots = \mu_{j-1,j}\beta_{j-1} = \mu_{j+1,j}\beta_{j+1} = \ldots = \mu_{n,j}\beta_n = \\ \mu_{1,j}\beta_j = \mu_{2,j}\beta_n = \ldots = \mu_{j-1,j}\beta_j = \\ \mu_{j+1,j}\beta_j = \ldots = \mu_{n,j}\beta_j = \rho_{1,j}\beta_1{}^4 = \rho_{2,j}\beta_2{}^4 = \\ \ldots = \rho_{j-1,j}\beta_{j-1}{}^4 = \rho_{j+1,j}\beta_{j+1}{}^4 = \ldots = \\ \rho_{n,j}\beta_n{}^4 = \rho_{1,j}\beta_j{}^4 = \rho_{2,j}\beta_j{}^4 = \ldots = \rho_{j-1}\beta_{j-1}{}^4 = \\ \rho_{j+1,j}\beta_{j+1}{}^4 = \ldots = \rho_{n,j}\beta_n{}^4 \qquad j = 1, \ldots, n \quad (4) \end{gathered}$$

Since the description of the complete mathematical problem for the system of normal differential equations (1) includes a set of initial conditions as those necessary and sufficient for the existence and uniqueness of a solution to the equations,[8] it must be determined whether or not the preceding is consistent with these initial conditions and whether the initial conditions yield any additional information about similarity. The initial conditions are that at $t = t_0$, $T_j = b_j$, where b_j are particular values of T_j at zero time. These conditions, together with Eqs. (3), give

$$t_0 = \omega t_0{}^* \quad (5)$$

and

$$b_j = \beta_j b_j{}^* \quad (6)$$

Now, without loss of generality, if $t_0 = 0$, then $t_0{}^* = 0$, since $\omega \neq 0$. Therefore, at the start of a transient test on a scale model, the initial temperatures on the model can be used to determine the corresponding initial temperatures on the prototype. Furthermore, the start of the test can be considered zero time for the model corresponding, in turn, to zero time for the prototype. Thus, the initial conditions offer no particular difficulty, but do not yield any useful additional information.

Relations (4), then, are the necessary and sufficient conditions for complete thermal similarity between the model and the prototype. These conditions, together with (3), yield 28 ratios that must be constant from prototype to model if complete similarity is to be obtained. However, all of these 28 ratios are not independent. Several independent sets of ratios could be deduced from these, and one set that seems to be the most interesting is given in Table 1.

It is interesting to note that the six ratios shown in Table 1 could almost be obtained by inspection of the six terms in the original differential equations (1), giving due consideration to the time derivative. This amounts to recognition of the relative simplicity of the differential equations. It also supports the intuitive choice for these particular ratios to constitute the independent set. Other independent sets

Table 1 A set of independent similarity ratios

$\dfrac{T_j}{T_k} = \dfrac{T_j{}^*}{T_k{}^*}$	(a)
$\dfrac{E_{kj}F_{kj}A_k}{\epsilon_j A_j} = \dfrac{E_{kj}{}^*F_{kj}{}^*A_k{}^*}{\epsilon_j{}^*A_j{}^*}$	(b)
$\dfrac{\alpha_j{}^SA_jSt}{C_jT_j} = \dfrac{\alpha_j{}^{*S}A_j{}^*S^*t^*}{C_j{}^*T_j{}^*}$	(c)
$\dfrac{\epsilon_j\sigma A_jT_j{}^3t}{C_j} = \dfrac{\epsilon_j{}^*\sigma A_j{}^*T_j{}^{*3}t^*}{C_j{}^*}$	(d)
$\dfrac{C_{kj}t}{C_j} = \dfrac{C_{kj}{}^*t^*}{C_j{}^*}$	(e)
$\dfrac{q_jt}{C_jT_j} = \dfrac{q_j{}^*t^*}{C_j{}^*T_j{}^*}$	(f)

could be found from all those implied in Eq. (4). In writing ratio (6b), it has been assumed that

$$R_{kj} = E_{kj}F_{kj}A_k\,\sigma \tag{7}$$

where F_{kj} is the classical geometry factor based on diffuse radiation and E_{kj} is a factor that depends on only the emissivities of the surfaces for bodies k and j.

Comparison of Results with Other Investigators

Several investigators have published results of dimensional analysis of some thermal problems of space vehicles.[9–12] Clark, Laband, and Katz give three dimensionless ratios that must be constants:

$$\frac{Tk}{qL^2} \qquad \frac{T^3L\sigma}{k} \qquad \frac{tk}{CL^2} \tag{8}$$

These ratios assume one characteristic length and were used to study the thermal response of composite wall constructions.

Vickers lists the eight dimensionless ratios shown in the first column of Table 2. The corresponding ratios obtained from Eq. (4) are shown in the second column of Table 2. The third column shows the equivalence between the first and second columns, under the assumptions listed in the

Table 2 Comparison with results of other investigators

JPL RATIOS	CORRESPONDING MSFC RATIOS	MSFC RATIOS IN JPL NOMENCLATURE	REMARKS
$\frac{q'''_m L^2_m}{k_m T_m} = \frac{q'''_p L^2_p}{k_p T_p}$	$\frac{C_{kj}T_j}{q_j} \quad \frac{C^*_{kj}T^*_j}{q^*_j}$	$\frac{q''' L^2}{kT} = \frac{q'''^* L^{*2}}{k^*T^*}$	$C_{kj} = \frac{kA}{L} = kL,\ q = q'''L^3$ where q''' is watts per unit volume
$\frac{(\rho C_p)_m L^2_m}{k_m t_m} = \frac{(\rho C_p)_p L^2_p}{k_p t_p}$	$\frac{C_{kj}t}{C_j} = \frac{C^*_{kj}t^*}{C^*_j}$	$\frac{(\rho C_p)L^2}{kt} = \frac{(\rho C_p)^* L^{*2}}{k^* t^*}$	$C_{kj} = kL,\ C_j = \rho C_p L^3$
$\frac{\epsilon_m \sigma T^3_m L_m}{k_m} = \frac{\epsilon_p \sigma T^3_p L_p}{k_p}$	$\frac{R_{kj}T^3_j}{C_{kj}} = \frac{R^*_{kj}T^{*3}_j}{C^*_{kj}}$	$\frac{\epsilon \sigma T^3 L}{k} = \frac{\epsilon^* \sigma T^{*3} L^*}{k^*}$	$C_{kj} = kL,\ R_{kj} = \epsilon\sigma L^2$ where $F_{kj} = F^*_{kj}$
$\frac{C_m L_m}{k_m} = \frac{C_p L_p}{k_p}$	C_m in this equation is the thermal contact conductance, watts/m^2 °K. The differential equations from which MSFC ratios were derived did not include a joint conductance term explicitly. See Note.		
$\frac{\alpha_{sm} S_m L_m}{k_m T_m} = \frac{\alpha_{sp} S_p L_p}{k_p T_p}$	$\frac{C_{kj}T_j}{{}^sA\alpha S} = \frac{C^*_{kj}T^*_j}{{}^sA^*\alpha^* S^*}$	$\frac{\alpha SL}{kT} = \frac{\alpha^* S^* L^*}{k^*T^*}$	$C_{kj} = kL,\ {}^sA = L^2$
$\frac{\alpha_{ijm} L_m \Phi_{jim}}{k_m T_m} = \frac{\alpha_{ijp} L_p \Phi_{jip}}{k_p T_p}$	$\frac{R_{kj} T^4_j}{C_{kj} T_k} = \frac{R^*_{kj} T^{*4}_j}{C^*_{kj} T^*_k}$	$\frac{\alpha_{kj} L \Phi_{jk}}{kT_k} = \frac{\alpha^*_{kj} L^* \Phi^*_{jk}}{k^*T^*_k}$	$C_{kj} = kL,\ R_{kj} = \epsilon_k \epsilon_j A_j F_{jk}$ which obtains $\frac{\epsilon_k \epsilon_j A_j F_{jk} \sigma T^4_j}{kLT_k} = \frac{\epsilon^*_k \epsilon^*_j A^*_j F^*_{jk} \sigma T^{*4}_j}{k^*L^*T^*_k}$ then let $\epsilon_k = \alpha_{kj},\ A_k = L^2$ and $\Phi_{jk} = \frac{\epsilon_j A_j F_{jk} \sigma T^4_j}{Ak}$
$\frac{q_m}{L_m k_m T_m} = \frac{q_p}{L_p k_p T_p}$	$\frac{q_j}{C_{kj}T_j} = \frac{q^*_j}{C^*_{kj}T^*_j}$	$\frac{q}{LkT} = \frac{q^*}{L^* k^* T^*}$	$C_{kj} = kL$
$\frac{q'''_m L_m}{k_m T_m} = \frac{q'''_p L_p}{k_p T_p}$	$\frac{q_j}{C_{kj}T_j} = \frac{q^*_j}{C^*_{kj}T^*_j}$	$\frac{q'' L}{kT} = \frac{q''^* L^*}{k^*T^*}$	$C_{kj} = kL,\ q_j = q''L^2$ where q'' is watts per unit area

Note: One of the C_{kj} terms can be interpreted as a joint conductance term. If this is done and compared with a straight conduction term, the following applies: The joint term is of the form $K_cA_c/L_c(T_k - T_j)$, where K_c is the effective contact conductivity, A_c and L_c being the corresponding effective area and length, respectively. Define $C = \left(\frac{K_cA_c}{L_c}\right) \times \left(\frac{T_k - T_j}{\bar{T}_k - \bar{T}_j}\right)$ Let $T_k - T_j = \bar{T}_k - \bar{T}_j$ and $C_{kj} = kA/L$, then $C = K_c/L_e$, w/m$^3\Delta$°K, and the appropriate ratio is $\frac{K_cA_cT_j/L_c}{kAT_j/L} = \frac{K_c^*A_c^*T_j^*/L_c^*}{k^*A^*T_j^*/L^*}$ or $\frac{CA_cT_j}{kAT_j/L} = \frac{C^*A_c^*T_j^*}{k^*A^*T_j^*/L^*}$ or $\frac{CL}{k} = \frac{C^*L^*}{k^*}$ where $A_c = A$.

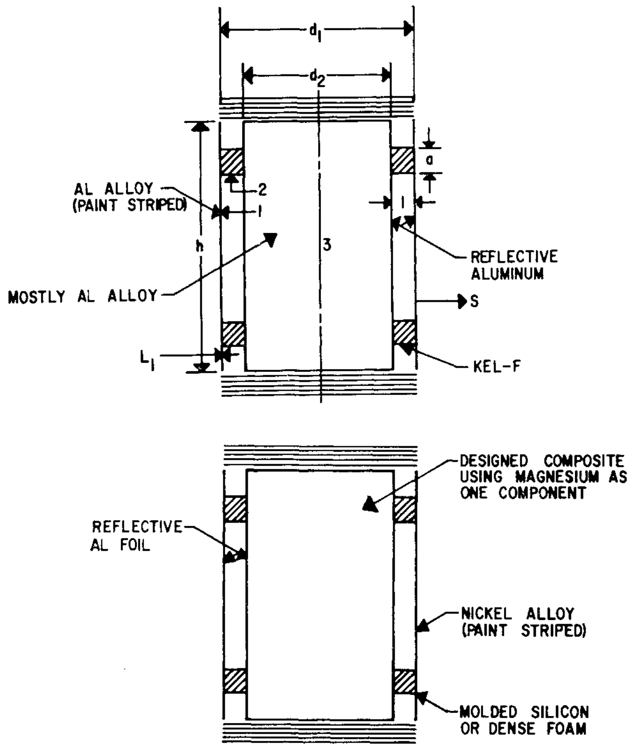

Fig. 1. Concentric cylinders with insulation.

remarks.‡ Several points are worth mentioning. It is seen that three of the ratios in the first column depend upon the same ratio, shown three times in the second column (the first and last two ratios listed). Consequently, these three ratios are not independent, leaving six ratios. One of these (the fourth listed) is a special ratio that accounts for the energy transfer across a structural joint, the so-called contact problem. This problem is the subject of considerable research effort and cannot be given adequate attention here. Vickers also reports on experimental work done under an Arthur D. Little Company/NASA Contract. The reader is referred to the report for the details.[11] Briefly, it involves the steady-state case.

Wainwright and his UCLA associates exhibit groups of three independent ratios based upon principles of dimensional analysis, assuming seven parameters bearing on the problem.[12] One of the groups is identical to those given by Katz (Eqs. 8). They also present an analysis based on the one-dimensional Fourier conduction equation with a radiation boundary condition, which is then adapted to a spacecraft thin wall, free to exchange radiant energy with a source and a sink. Five ratios are obtained for the case in which surfaces are coupled by radiation and three if there is not radiative coupling. The Wainwright group also discuss some of the difficulties involved when external energy is supplied to the models by solar simulation showing, by example, that in some situations the equivalent intensity of 20 suns is required. An arbitrary reference temperature is defined which may be manipulated to circumvent some of these difficulties. This technique can be related to ratio (6a), T_k/T_j (Table 1).

In the preceding discussion in this section, only one characteristic length has been considered. This could impose a severe limitation on the modeling feasibility, since its effect is to require strict geometric similarity between the model and prototype, thus requiring adjustments in material properties, temperature correspondence, and source intensity. It tends to preclude geometric distortion of the model, which may be desirable as a technique to avoid search for nonexistent material properties. In particular, the characteristic lengths involved in radiative transfer, through the geometric factor F_{kj}, Eq. (7), may need to be identified separately from the lengths involved in the conduction coefficient. In this way, thin shells would not be made thinner by scaling, and insulations between outside shells and internal components might not be scaled down as much as large radiative areas and over-all lengths. This will be discussed later in more detail.

Investigators at Lockheed Missiles and Space Company have made the only other analysis[13] of thermal similitude based upon the differential equations used in the thermal design of space vehicles which is known to the present author. They recognize also the possibility of distorted geometry in the conducting path as an attractive modeling technique.

To summarize this short comparison, there is considerable activity in investigations of the feasibility of thermal modeling. The investigations are sufficiently independent to retain a degree of originality but still closely enough related to allow for comparisons to be made. There are many problems to solve and techniques to develop if modeling is to be proved feasible. The author believes that it is desirable to encourage several approaches so that all promising aspects will be investigated; otherwise, a feasible method might be left undiscovered.

Example

Shown in Fig. 1 is a cylindrical instrument compartment, enclosed by a concentric cylindrical shell, with insulating support rings between the shell and instrument section. Normally, the facing surfaces of the cylinders would have low emissivities for insulation in the radiation mode. The external surfaces of the shell would have carefully chosen solar absorptivity and infrared emissivity to give the proper thermal balance with the external radiation environment. The ends of the cylinders are considered to be adiabatic, for simplicity. The angle between the centerline and the vector to the sun is 90°. Data needed for calculating the similarity ratios are also given in the figure. In practice, only two differential equations would be written for this case, one for the shell and one for the instrument unit. Various factors in the differential equations are as follows:

$$
\begin{array}{ll}
F_{13} = 1 & {}^{S}A_1 = d_1h \\
F_{12} = 0 & {}^{S}A_2 = 0 \\
F_{23} = 0 & {}^{S}A_3 = 0 \\
F_{32} = 0 & A_1 = \pi d_1 h \\
F_{21} = 0 & A_2 = 0 \\
F_{31} = 1 & A_3 = 0 \\
V_1 = \pi d_1 h L & C_1 = (\rho C_p)_1 V_1 \\
V_2 = (\pi/2)(d_1^2 - d_2^2)a & C_2 = (\rho C_p)_2 V_2 \\
V_3 = (\pi/4)d_2^2 h & C_3 = (\rho C_p)_3 V_3 \\
{}^{R}A_1 = \pi d_1 h & C_{31} = C_{13} = k_2{}^{C}A_2/l \\
{}^{R}A_2 = 0 & C_{23}, C_{32}, C_{12}, C_{21} \\
{}^{R}A_3 = \pi d_2 h & \text{do not exist}
\end{array}
$$

$$E_{31} = E_{13} = \frac{1}{\dfrac{1}{\epsilon_3} + \dfrac{1}{\epsilon_1} - 1} \qquad \begin{array}{l} E_{21}, E_{12}, E_{23}, E_{32} \\ \text{do not exist} \end{array}$$

The ratios given in Table 1 will be examined.

$$\frac{T_k}{T_j} \quad \text{implies} \quad \frac{T_1}{T_3} \quad \text{or} \quad \frac{T_3}{T_1} \tag{9a}$$

‡ It is not intended that the assumptions used for comparison of the results are those that should be used for model design.

$$\frac{E_{kj}F_{kj}{}^R A_k}{\epsilon_j A_j} \quad \text{implies} \quad \left.\begin{aligned} &\frac{E_{31}F_{31}{}^R A_3}{\epsilon_1 A_1} = \left(\frac{d_2}{d_1}\right)\left(\frac{\epsilon_3}{\epsilon_1 + \epsilon_3 - \epsilon_1\epsilon_3}\right) \\ &\frac{E_{13}F_{13}{}^R A_1}{\epsilon_3 A_3} = \left(\frac{d_1}{d_2}\right)\left(\frac{\epsilon_1}{\epsilon_3 + \epsilon_1 - \epsilon_3\epsilon_1}\right) \end{aligned}\right\} \quad (9b)$$

$$\frac{\alpha_j{}^S A_j St}{C_j T_j} \quad \text{implies} \quad \frac{\alpha_1{}^S A_1 St}{C_1 T_1} = \frac{\alpha_1 St}{(\rho c_p)_1 T_1 L_1 \pi} \quad (9c)$$

$$\frac{\alpha_j A_j T_j{}^3 t}{C_j} \quad \text{implies} \quad \frac{\epsilon_1 \sigma A_1 T_1{}^3 t}{C_1} = \frac{\epsilon_1 \sigma T_1{}^3 t}{(\rho c_p)_1 L_1} \quad (9d)$$

$$\frac{C_{kj}t}{C_j} \quad \text{implies} \quad \left.\begin{aligned} &\frac{C_{31}t}{C_1} = \frac{k_2(d_1 - d_2)at}{(\rho c_p)_1(d_1 h L_1)l_2} \\ &\frac{C_{13}t}{C_3} = \frac{k_2(d_1 - d_2)at}{(\rho c_p)_3\left(\frac{d_2{}^2 h}{4}\right)l_2} \end{aligned}\right\}$$

In these last two relations, it is convenient to rearrange them into two factors each:

$$\left.\begin{aligned} &\frac{k_2 t}{(\rho c_p)_1(L_1 l_2)} \quad \text{with} \quad \frac{a}{h}\left(1 - \frac{d_2}{d_1}\right) \\ &\frac{k_2 t}{(\rho c_p)_3(d_2 l_2)} \quad \text{with} \quad 4\left(\frac{a}{h}\right)\left(\frac{d_1}{d_2} - 1\right) \end{aligned}\right\} \quad (9e)$$

$$\frac{q_j t}{C_j T_j} \quad \text{implies} \quad \left.\begin{aligned} &\frac{q_1 t}{C_1 T_1} = \frac{q_1 t}{(\rho c_p)_1 T_1(\pi d_1 h L_1)} \\ &\frac{q_3 t}{C_3 T_3} = \frac{q_3 t}{(\rho c_p)_3 T_3(\pi/4 d_2{}^2 h)l_2} \end{aligned}\right\} \quad (9f)$$

Consider the problem of length in these ratios. Only the temperatures are involved in (9a). In (9b), lengths occur as ratios, so that scaling is not a problem. In Eqs. (9c) and (9d) only the shroud thickness occurs, so that there is clearly one length. The ratios (9e) have been so arranged that the auxiliary ratios involve only ratios of lengths, leaving in the main ratios the product of two other lengths. If one characteristic length is now chosen for (9e), it will represent in one case $(L_1 l_2)^{1/2}$ and in the other $(d_2 l_2)^{1/2}$. Now l_2 occurs in both, and the remaining lengths are the shroud thickness and the inner cylinder diameter. Any net change in the products $L_2 l_2$ or $d_2 l_2$ will require a change in material properties. This ratio appears to offer some difficulty in scaling. The ratios (9f) seem even more complicated if one characteristic length is chosen; however, the internal energies will be relatively easy to scale so that the difficulties caused by the lengths can be offset. Since it is desirable to avoid throwing all of the burden of scaling on material properties, geometrically distorted models are indicated.

If we let the shroud thickness L_1 and the conductive length l_2 remain unchanged while all other linear dimensions are reduced by a factor of $\frac{1}{4}$, then ratios (9a), (9b), (9c), (9d), and the first of (9e) remain unchanged. In the ratios of (9f), the internal generation would be reduced, q_1 by a factor of $\frac{1}{16}$ and q_3 by a factor of $\frac{1}{64}$. (These factors may be so small as to make these last ratios insignificant.) The last of the ratios (9e) requires that either $(\rho c_p)_3$ changes by a factor of 4 or k_2 changes by $\frac{1}{4}$. Inspection of available material properties normally used in space vehicles shows that the following ranges apply for the more useful materials§:

$$\begin{aligned} 0.3 \leq (\rho c_p)_N \leq 2.5 \qquad &\text{(plastic foam, 11.0)} \\ 0.14 \leq k_N \leq 5.5 \qquad &\text{(Cu, hard drawn, 1668)} \end{aligned} \quad (10)$$

If *all* linear dimensions are scaled by a factor of $\frac{1}{4}$, the situation is worse, even if $\frac{1}{4}$ scale is a fairly mild requirement. It is clear that a factor of 4 for (ρc_p) is not readily available. A factor of $\frac{1}{4}$ for k_2 appears to be available only in the plastic foams. If the insulating rings were made of KEL-F, which would be likely in thermal design practice, then it would be possible to substitute a plastic foam ring as a model substitute having $\frac{1}{4}$ thickness and the same conductive length as the prototype. However, if k_2 is changed in one ratio (9e) where the lengths change, it also changes in the other ratio (9e), where the lengths did not change. It appears that it is necessary to change both (ρc_p) and k. Let k_2 be changed by a factor of $\frac{1}{2}$, using molded silicone or relatively dense plastic foam. Let $(\rho c_p)_1$ be changed by $\frac{1}{2}$ and $(\rho c_p)_3$ be changed by 2 by using a designed composite for $(\rho c_p)_3$ and a nickel alloy for $(\rho c_p)_1$. This appears to be possible in view of the material property data. (Tin has the right ratio but is not structurally rigid and might give trouble because of its low melting point; magnesium alloy appears to be one of the materials to use in the composite.) Now, change ϵ_1 in (9d) by a factor of $\frac{1}{2}$ to offset the change in $(\rho c_p)_1$. This is the emissivity of the *external* surface of the shroud, for which such a change could be accomplished easily by the use of paint stripes on a portion of the external area of the nickel alloy shroud. Since the materials of the shroud and instrument unit have changed, the internal surface of the shroud and external surface of the instrument unit have changed, which affects ratio (9b). However, these surfaces could be overlaid with thin aluminum foil, attached well, and, thus, retain the prototype emissivities in ratio (9b). This leaves ratio (9c), which must be adjusted for the change in $(\rho c_p)_1$. There are three possibilities: α_1 can change by $\frac{1}{2}$, S can change by $\frac{1}{2}$, or both can change such that the product of the changes is $\frac{1}{2}$. Now α_1 has actually already changed because of the change in material and in ϵ_1. If α_1/ϵ_1 were about 1.5 on the prototype, which would correspond to sandblasted Al 2024 ($\alpha = 0.67$, $\epsilon = 0.30$) surface with white paint stripes ($\alpha = 0.2$, $\epsilon = 0.9$), the change to a nickel shroud with aluminum foil overlay ($\alpha = 0.19$, $\epsilon = 0.04$) would require a darker paint for model stripes ($\alpha = 0.05$, $\epsilon = 0.9$). In this case S would not change.

Table 3 summarizes the situation, which is shown schematically in Fig. 1. Thus, a thermal model appears feasible for this particular prototype for the transient case, with the temperature-time relationships being identical between the model and prototype, by using geometric distortion of the insulating rings and by changing the material properties within practical limits.

Table 3 Modeling results for the example

Parameter	Model/ prototype value
d_1, d_2, a, h	$\frac{1}{4}$
L_2, l_2	1
T_1, T_3	1
t	1
k_2	$\frac{1}{2}$
$(\rho c_p)_3$	2
$(\rho c_p)_1$	$\frac{1}{2}$
ϵ_1, ϵ_3 (internal)	1
ϵ_1 (external)	$\frac{1}{2}$
q_1	$\frac{1}{32}$
q_3	$\frac{1}{32}$
αS	$\frac{1}{2}$

§ The subscript N denotes normalization of the property for various materials to one particular material. The volumetric heat capacity was normalized to Al 6061 and the thermal conductivity to KEL-F.

It might be well at this point to review the purposes of full-scale thermal testing of space vehicles and to determine whether or not these same purposes can be met with a scaled model. In the past, one purpose of thermal testing was the experimental determination of conduction and radiation coefficients between various regions [that is, the C_{kj}'s and R_{kj}'s in Eqs. (1)] and time constants of the various regions under a sudden change of external environment. Solar simulation in spectral distribution and intensity was not used, and no attempt was made to simulate the shadow-sun cycles, since this was not necessary for the coefficient determinations. The experimentally determined coefficients were used as a check on the design concept and the calculated coefficients which were used as inputs in the computer solutions of the differential equations. If these tests were now run on the scale model, the experimental coefficients obtained would not be of value in checking the calculated coefficients of the prototype, since the model design was based upon recalculating thermally similar coefficients (in effect) and assuming the coefficients of the prototype to be known correctly. There is at least as much room for error in designing the model as there was in the design of the full-scale vehicle. This does not mean, necessarily, that this will be the case for every prototype considered, albeit the example considered was relatively simple.

If the basic purpose of the testing were the determination of the thermal response of the vehicle by complete simulation of the external environment, then the dilemma would be somewhat similar. With the material property changes between model and prototype, it would not be clear whether the model tests were checking the computer analysis or whether the computer analysis was checking the modeling. This is not meant to imply that modeling by way of complete thermal similarity does not make sense; rather, it is the intent to raise some basic questions. The many aspects to the problem of thermal modeling of space vehicles not yet examined must be studied thoroughly before a complete answer is attempted; otherwise, the answers will be premature and possibly incorrect. It is appropriate to note the great expense in building large simulation facilities. However, it may develop that thermal modeling has the greatest potential in analyzing some particular thermal aspect of a design—separately, without complete similarity—and in special problems in heat transfer.

An area where modeling is attractive is the experimental determination of radiation exchange coefficients. The technique here would be to break down complex, many-body problems into pairs of surfaces between which only radiation exchange is considered. All conductive paths between the surfaces would be left out of the model. The integral expression for the radiation geometry factor from a surface A_2 to a differential area dA_1 is[6]

$$F_{(d_1)2} = (1/\pi) \int_{\omega_2} \cos\phi_1 d\omega_2 \tag{11}$$

where ω_2 is the solid angle subtended by A_2 as viewed from dA_1, and ϕ_1 is the angle between normal to dA_1 and the line of sight to a differential area of A_2. It is seen from Eq. (11) that complete geometric scaling leaves the geometry factor between two model surfaces the same as that between the two prototypes. Therefore, if an experimental technique for determining geometry factors is available, modeling can be used to reduce or enlarge surfaces to convenient sizes for laboratory work.

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